How Accurately Do Structural Asymmetric First-Price Auction Estimates Represent True Valuations?

Kirill Chernomaz* and Hisayuki Yoshimoto†‡

August 15, 2019

Abstract

Structural asymmetric first-price auction estimation methods have provided numerous empirical studies. However, due to the latent nature of underlying valuations, the accuracy of estimates is not feasibly testable with field data, a fact that could inhibit empirical auction market designs and applications based on structural estimates. To assess their accuracy, we provide an analysis of estimates derived from experimental asymmetric auction data, in which researchers observe valuations. We test the null of statistical equivalence between the estimated and true value distributions against the alternative of non-equivalence. When advanced models are used, the Modified Kolmogorov-Smirnov test fails to reject the distributional equivalence, supporting structural asymmetric auction estimations for auction market studies. In addition, recovered efficiencies have plus-minus 2.5 percent precision, compared to the true efficiencies.

Keywords and Phrases:

Structural Auction Estimation, Semi/Nonparametric Estimation, Asymmetric Auction, Risk Aversion

JLE Classifications:

C13, D44

*San Francisco State University, Department of Economics, kirillch@sfsu.edu

†University of Glasgow, Adam Smith Business School, Hisayuki.Yoshimoto@glasgow.ac.uk

‡A version of this paper was presented at the 2014 Industrial Organization: Theory, Empirics, and Experiments Conference in Alberobello (Italy); 2014 European Association for Research in Industrial Economics conference in Milan; 2015 International Industrial Organization Conference in Boston; 2015 Auction, Competition, Regulation, and Public Policy Conference in Lancaster (UK); 2016 Royal Economic Society Conference in Sussex (UK); and 2017 Econometric Society Asian Meeting in Hong Kong. We are grateful for the comments from conference session discussants: Daniel Garcia, A.M. (Sander) Onderstal, and Paulo Somaini. We are also thankful for the insightful comments provided by Jorge Balat, Matthew Gentry, Emmanuel Guerre, and Timothy Hubbard. The paper was previously circulated under the title of "Are Estimates of Asymmetric First-Price Auction Models Credible? Semi & Nonparametric Analyses." Any errors are our own. The usual disclaimer applies.

1 Introduction

Structural studies of asymmetric auction markets based on estimated valuations have attracted many economic researchers, as well as auction market practitioners, yet the accuracy of estimated valuations is still unexplained. By investigating estimates derived from experimental auction data, in which researchers observe laboratory-assigned valuations, and by statistically testing the equivalence between estimated and observed valuations, this study contributes to the literature by reporting the performance of asymmetric first-price auction estimates based on semi and nonparametric methods.

The empirical and structural study of auction markets for allocating scarce resources is a successful area in economics; numerous empirical findings on auction theory have been reported in the last twenty years. In the literature, researchers are interested in understanding strategic interactions among bidders for designing auction markets based on economic incentives. Initially, linear regression models were used, although their use for analyzing bidders' non-linear payoff-maximization problems was a challenge in describing strategic behavior. In order to overcome this difficulty, semi and nonparametric structural estimation methods arose and, for the last twenty years, have been widely used for investigating market-design implications.[1] One of the advantages is that researchers are also able to recover the structural elements of auction theory, such as bidders' private valuations and payoff functions. Accordingly, counterfactual market-design analyses, including market efficiency assessments, have become possible. In addition, due to the pervasiveness of asymmetry among bidders, the estimation methods have also been extended to asymmetric auctions. As a result, asymmetric auction estimates are now the vital foundation of auction market research for addressing positive and normative questions.

However, while more and more asymmetric first-price auction estimates are reported in the literature, there is a fundamental difficulty in evaluating the performance of these estimates. In structural studies, researchers use econometric methods based on theoretical auction models, in which they can use observed bids to estimate bidders' valuations, as valuations are not directly observed in empirical first-price auctions. The latent nature of bidders' valuations makes comparison between estimated and true valuations infeasible. In addition, most of the auction estimation methods provide few testable frameworks for enabling empirical researchers to examine the modeling assumptions. Such an infeasible comparison and the lack of testability could then foster divided views on empirical auction estimates. Specifically, there are two major kinds of adverse views. The

[1] The cornerstone of empirical and structural first-auction literature should be credited. To the best of our knowledge, the literature was initiated by the Ph.D. thesis of Paarsch (1992) with parametric models. Donald and Paarsch (1993) Donald and Paarsch (1996), Elyakime, Laffont, Loisel, and Vuong (1994), and Laffont, Ossard, and Vuong (1995) established statistically rigorous yet flexible parametric estimation methods. The survey of Hickman, Hubbard, and Sağlam (2012) concisely summarizes the literature, while that of Gentry, Hubbard, Nekipelov, and Paarsch (2018) provides a longer treatment of developments in the structural econometrics of auctions.

first is made by a group of robust mechanism design researchers who have a skeptical view on bidders' ability to find Bayesian Nash Equilibrium (BNE), especially in asymmetric auctions. The second view comes from a group of applied researchers who contend that assumptions made for structural analyses are implausibly strong.

Against such adverse views, Bajari and Hortaçsu (2005) provide a concrete and focused response by using *symmetric* first-price auction data from a laboratory study, in which researchers observe experimentally-assigned true valuations. By using the valuations assigned in the experiment as a benchmark, they compare the estimates generated by various structural models with semiparametric estimation methods. Their analyses show that estimates based on the Constant Relative Risk Aversion (CRRA) model can recover the distributions of latent valuations in symmetric auctions with a statistically acceptable degree of accuracy, at least in (but not limited to) a laboratory environment. This demonstrates the great potential of structural auction estimation methods. The current study contributes to the empirical auction literature by extending the symmetric first-price auction research of Bajari and Hortaçsu (2005) to asymmetric bidders and nonparametric payoff function estimation models.

By observing field auctions, we can find numerous situations where the symmetric auction assumption limits the scope of empirical investigations, and where asymmetric auction structural estimation models are essential. In fact, the majority of empirical auction studies reported for the last decade use asymmetric auction models for estimating latent valuations.[2] Such estimated asymmetric valuations are, in turn, used for deriving market design policy implications, supported by structural counterfactual simulations. Thus, examining the accuracy of estimated asymmetric valuations is vital for achieving policy goals, such as the improvement of market efficiencies and an increase in revenues.

Given the ubiquity of asymmetry among bidders in field auctions and the challenges reported in the development of theoretical asymmetric auction studies, it is our belief that investigating the precision of asymmetric auction estimates and suggesting improvements is a valuable contribution to the literature. Following the precedent set by Bajari and Hortaçsu (2005), we use laboratory data, which are able to provide insights on the quality of estimates.[3] Both strengths and shortcomings are directly gauged with laboratory data, and

[2]The incomplete list of the sources of asymmetries and studies are: *firm-size* (Marion 2007, also as indicated by Laffont et al. 1995), *joint bids* (Hendricks and Porter 1992 and Campo et al. 2003), *capacity constraints* (Jofre-Bonet and Pesendorfer 2003 and Balat 2012), *collusive behaviors* (Porter and Zona 1999, Pesendorfer 2000, Bajari and Ye 2003, and Asker 2010), *asymmetric information* (Hendricks and Porter 1988 and Hendricks et al. 1994), *bid preference* (Krasnokutskaya and Seim 2011, and aforementioned Marion 2007), *bidder experience* (De Silva et al. 2003 and Campo 2012), and *geographic locations* (Flambard and Perrigne 2006). In addition, Cantillon (2008) emphasizes the importance in modeling bidder asymmetry for investigating expected revenues.

[3]Ertaç, Hortaçsu, and Roberts (2011) and Salz and Vespa (2015) also use experimental data for structural estimations. Specifically, in the first study, the authors exploit a unique experiment structure, which includes both first and second price auctions, and investigate the entry behavior into auctions. In the second study, the authors evaluate parameter recoveries and counterfactual

such findings are essential for extending the capability of asymmetric auction market studies. In addition, to the best of our knowledge, a direct evaluation of asymmetric auction estimates has not been previously reported in the literature.

Specifically, by exploiting the experimental data and by comparing the true and estimated valuations, we evaluate auction estimates, which are inevitably affected by a combination of factors: (1) bidders' ability to behave as prescribed by the BNE and (2) the flexibility of econometric methods which are built upon the BNE. (1) and (2) are distinguishable only when we assume full rationality among all bidders, which may not be guaranteed in empirical studies. However, our experimental comparisons reveal that the proposed auction estimation methods are capable of recovering true valuations, even without a full rationality assumption, and with asymmetry among bidders.

Moreover, there are behavioral concerns regarding the asymmetric framework. The asymmetric auction theory places multiple requirements on bidders' behavior and their cognitive ability: each bidder not only behaves in a payoff-maximizing rational manner but also recognizes that rival bidders behave in the same fashion. Furthermore, bidders understand rival bidders' asymmetric bidding behavior and make asymmetric best responses, finding an asymmetric fixed point in a functional space. Given the existence of behavioral bidders, who may not follow the theoretical prescription and the intricacy of asymmetric best responses, a deliberate investigation is required for verifying whether estimated asymmetric valuations closely approximate true valuations.

In this study, we look at the performance of asymmetric first-price auction estimation methods introduced by Isabelle Perrigne, Quang Vuong, and their co-authors. Due to their versatility in allowing asymmetry in value distributions and computational tractablity, these methods are now the standard used by empirical works for investigating auction markets. We study the precision of estimates derived by the semi and nonparametric estimation methods for asymmetric auctions with an exogenous variation in an auction environment.[4]

Specifically, we use a dataset from the asymmetric private value first-price auctions experiment conducted by Chernomaz (2012). The data contains bids and laboratory-assigned valuations for each bidder. Additionally, Chernomaz (2012) investigates the effects caused by asymmetry among bidders under exogenously changing auction environments, while the majority of bidder valuations remain fixed before and after such exogenous

predictions of Markov-perfect dynamic oligopoly models.

[4]The history is as follows: based on the cornerstone work of Guerre, Perrigne, and Vuong (2000) that proposes a nonparametric method for symmetric first-price auctions with risk neutral bidders, Campo, Perrigne, and Vuong (2003) extend the nonparametric estimation method to asymmetric auctions. In addition, Guerre, Perrigne, and Vuong (2009) and Campo, Guerre, Perrigne, and Vuong (2011) broaden the estimation method to allow risk averse preferences among bidders with semi and nonparametric payoff functions.

changes. Our estimation strategy takes advantage of the exogenous changes in auction environments to identify bidders' payoff functions and underlying value distributions, as bid distributions vary before and after the exogenous change, while the valuations that bidders hold remain unchanged. We construct, then estimate, the *compatibility conditions* derived by connecting first-order conditions of the payoff maximization problems before and after the exogenous auction environment change.

We test the null hypothesis of accurate valuation estimates that are derived from structural asymmetric auction models (using various estimation methods of payoff functions) against the alternative of inaccurate valuations. Distributional equivalence tests compare the true and structurally estimated value distributions. The primary analytic methodology employed in our research follows Bajari and Hortaçsu (2005). However, we extend their analyses in two empirically important ways.[5] The first extension is that we investigate *asymmetric* value distributions among bidders and associated market efficiency. The second extension is that, in addition to the semi-parametric models, we investigate the recently developed nonparametric payoff functions that allow the greatest degree of flexibility in modeling of bidders' risk preferences.

Our study is different from, but complements Monte-Carlo simulation evaluations. Using simulated bids is an established approach for testing the validity of those estimation methods. The limitation of simulations is that all sources of uncertainty and all parameters are controlled by researchers, and full rationality of bidders is built into the simulation. Thus, the usefulness of the results to empirical studies is limited by the ability of the researcher to imitate human error. On the other hand, experimental data offers an additional opportunity where some aspects are controlled (i.e. valuations) while other aspects are not controlled (i.e. risk preferences and the degree of rationality in behavior). Experimental data is much noisier with uncontrollable error distributions, compared to simulated data. Using data generated by human behavior puts these methods to a more challenging test. Exposing the methods to such noisy environments helps us better understand the strengths and limitations of these proposed econometric methods. We show that we can still recover a valuation distribution close enough to the true one, despite these challenges. We also show that it is not a straightforward task, and that only after estimating the shape of the utility function are we able to achieve the result. The empirical implication is that these econometric methods can be used in applications where the full rationality of bidders is debatable. However, we show that an additional source of exogenous variation can be a great help for the successful recovery of valuations.

Based on comparisons between estimated and true private valuations, we report these main conclusions: (1) the risk neutral model assumption, which is often made for simplicity in the literature, tends to inflate

[5] Bajari and Hortaçsu (2005) also conduct the analyses of Adaptive Learning and Quantal Response Models. We exclude these models from this research as they have rarely been used in empirical auction literature, although we recognize that these models have intriguing aspects for understanding bidding behavior.

estimated valuations in an asymmetric auction environment; (2) the assumption of risk averse bidders is indispensable in empirical asymmetric auction research, as it enables nonnegligible improvements in the accuracy of estimates; (3) among semi and nonparametric models with risk aversion, the nonparametric model with shape restrictions based on conventional wisdom provides the most accurate results;[6] (4) when advanced risk averse models are employed, the Modified Kolmogoro-Smirnov test fails to reject the statistical equivalence between the estimated and true value distributions, supporting the findings reported in the empirical asymmetric auction literature; and (5) derived market efficiency based on structural estimates falls in plus-minus 2.5 percent range from the true efficiency, indicating the potential of structural asymmetric auction estimates for assessing market efficiency.

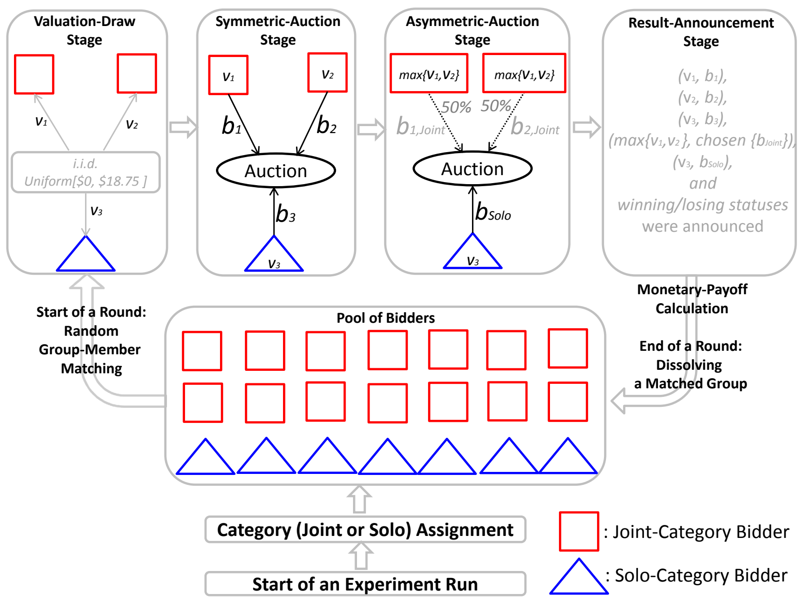
Lastly, the inherent limitations of our research should be noted. As our research uses data from experimental auctions, bidders' behavior could potentially be different from that observed in field auctions. Consequently, our results should be interpreted with caveats regarding the difference between field and experimental auctions.[7] On this point, we will provide detailed explanations of external validity in the conclusion section. Also, the cause of asymmetry investigated in this research is joint bidding, one of the many forms of asymmetries reported in the literature, and cautious interpretation is required for extending our results to other forms of asymmetric auctions. These limitations notwithstanding, the findings reported in this research provide support for the empirical analyses of asymmetric auction markets that are recently reported in the literature.

The rest of this article is organized as follows: Section 2 describes the experimental data that contains both valuation and bid observations; Section 3 explains the theoretical auction models that are the basis for the structural estimations; Section 4 describes the semi and nonparametric asymmetric auction estimation methods that are used to generate estimates; Section 5 provides the estimation results, reports the results of distributional equivalence tests, and describes recovered market efficiency; and lastly, Section 6 discusses the external validity and provides conclusions.

[6]We employ the nonparametric sieve estimation method with shape restrictions based on commonly reported experimental findings in the literature.

[7] Specifically, Kessler and Vesterlund (2015) argue that it is widely accepted, even among critics, that laboratory experiments tend to have qualitative external validity. On the other hand, researchers need to be cautious in interpreting specific quantitative findings of lab experiments to have broad external validity. Nevertheless, Kessler and Vesterlund (2015) also note that there is value in quantitative estimates if they are used as an intermediate step to facilitate investigations of qualitative regularities.

Figure 1: Stages within a Round



Note: A bidder category is assigned at the beginning of the experiment run, and a participant never changed his/her category throughout an experiment run.

Table 1: Numbers of Bidders and Bids

	Bidder Category	Number of Participants	Symmetric-Auction Stage Bids	Asymmetric-Auction Stage Bids
Experiment Run I	Joint	16 Bidders	192 Bids	192 Bids (96 chosen-then-announced bids 96 compatibility-condition-adapted bids)
	Solo	8 Bidders	96 Bids	96 Bids
Experiment Run II	Joint	12 Bidders	144 Bids	144 Bids (72 chosen-then-announced bids 72 compatibility-condition-adapted bids)
	Solo	6 Bidders	72 Bids	72 Bids
Aggregated Run (Run I and II)	Joint	28 Bidders	336 Bids	336 Bids (168 chosen-then-announced bids 168 compatibility-condition-adapted bids)
	Solo	14 Bidders	168 Bids	168 Bids

2 Data Descriptions

In this section, we illustrate the laboratory auction data used to obtain the results described in the empirical section. The key feature of the data is that bidders in experiments participated in two exogenously varying auction formats while the majority of their valuations remained unchanged. With the emphasis on such exogenous change, we first describe the laboratory auction procedures, then explain the summary statistics for illustrating differences in bidding behavior before and after the exogenous auction format change.

The data is from Chernomaz (2012), which investigates the results of joint bids in independent private value first-price auctions.[8] The participants were undergraduate students at the Ohio State University and were paid a \$6 show-up fee. The participants were from a wide range of social science majors and did not have much previous experience in auction bidding: a detailed description of auction procedures was provided to participants, as described below.[9]

A computer-based laboratory was used for this experiment, and participants interacted exclusively through computer screens. By using random computer-generated numbers, valuations of an object were exogenously and randomly assigned to bidders. At the beginning of each experiment run, bidder categories (joint and solo) were randomly assigned, and every participant remained the same category (either joint or solo) throughout the experiment run. We define a solo-category bidder as one who was not allowed to change valuation in the transition between symmetric- and asymmetric-auction stages within a round. On the other hand, we define a joint-category bidder as one who aggregated valuations (with the other joint-category bidder in a matched group) before an asymmetric-auction stage mid-round. At this point, participants were provided with a detailed explanation of experiment procedures, including: value distribution, valuation-drawing method, group-matching rule, valuation-aggregation method for joint-category bidders in an asymmetric-auction stage, auction rules, result-announcement timing and contents, winner determination rule, monetary payoff methods, and, crucially, the unchanged nature of bidder categories.

The structure of the entire experiment was as follows: there were two experiment runs (I and II) conducted on different days, and the participants were not allowed to join more than one experiment run.[10] In each experiment run, there were two practice rounds without monetary payment and twenty-four rounds with monetary payments. In each round, there were several stages, including symmetric- and asymmetric-auction stages, as described in Figure 1. The format was therefore (an experiment run) $\supset$ (a round) $\supset$ (a stage). Table 1 summarizes the number of participating bidders and bids used in this research in each experiment run.

[8] A joint bid (also known as a consortium bid) is defined as two or more bidders who form a group and submit one joint bid in an auction. Joint bids were allowed in Mexico and Louisiana Gulf Outer Continental Shelf (OSC) wildcat auctions, and as a result the implications of joint bids are now intensively investigated in the empirical auction literature, e.g. Hendricks and Porter (1988), Campo, Perrigne, and Vuong (2003), and Hendricks, Pinkse, and Porter (2003). Note that our laboratory procedures in Figure 1 can be viewed as a miniature hypothetical wildcat auction, in which both (non-collusive) individual and joint bids are permitted and the auctioneer randomly determines whether or not to allow a joint bid. One of the conclusions in Chernomaz (2012) is that qualitatively bidders follow theoretical predictions by changing their bidding behavior between symmetric and asymmetric environments. Additionally, bidders with the weaker (solo) valuation distribution tend to be pointwise more aggressive, as the theory predicts.

[9] It is worth emphasizing that risk attitudes were not elicited as part of the experiment. However, the design of the experiment allows us to estimate the revealed risk attitudes by comparing bids for the same valuation but in different bidding settings: symmetric vs. asymmetric.

[10] Chernomaz (2012) conducted one more experiment run. However, in the middle of that run, one subject elected to leave and had to be replaced with a substitute subject. To avoid any resulting issues, such as discrepancies in bid distributions and bidders' risk averse attitudes in structural estimations, we omit this experiment run from our analyses. See page 708 of Chernomaz (2012) for details.

At the beginning of each round, participating bidders were randomly matched to form three-bidder groups. Within a group, two bidders were joint-category players, and the remaining one was a solo-category player. Then, valuations were drawn from the uniform distribution for each bidder, $v_i \overset{\text{i.i.d.}}{\sim} F_V(v_i) = \frac{1}{18.75}v_i$, where $v_i \in [\$0, \$18.75]$, denoted by v_1 for a joint-category bidder; v_2 for another joint-category bidder; and v_3 for a solo-category bidder, as depicted in Figure 1.

As mentioned above, there were symmetric- and asymmetric-auction stages within each round. In the symmetric-auction stage, three bidders submitted one bid each (denoted as b_1, b_2, and b_3), though the outcome of a symmetric-stage auction was not announced until the result-announcement stage.

Next, at the beginning of an asymmetric-auction stage, the two joint-category bidders aggregated their valuations as $\max\{v_1, v_2\} = v_{\text{Joint}}$. Accordingly, valuations among joint-category bidders are distributed as $\max\{v_1, v_2\} = v_{\text{Joint}} \overset{\text{i.i.d.}}{\sim} F_{V_{\text{Joint}}}(v_{\text{Joint}}) = \frac{1}{(18.75)^2}v_{\text{joint}}^2$ where $v_{\text{Joint}} \in [0, 18.75]$. This method of value aggregation, adopting a maximum valuation among joint-category bidders, is motivated by empirical observations that joint bidders share their economic resources. Such resources include: the best available cost-saving technology; the closest geographical locations; and information on the best-available resale opportunities.

In an asymmetric-auction stage, a solo-category bidder submitted a bid b_{Solo}, based on his valuation of v_3. On the other hand, each joint-category bidder submitted a respective bid, based on the aggregated valuation of $v_{\text{Joint}} = \max\{v_1, v_2\}$. At this point, the two joint-category bidders were informed of their aggregated valuation (i.e. $\max\{v_1, v_2\}$) through their respective computer screens. However, verbal or textual communication between joint-category bidders was forbidden. Therefore, a bid made by a joint-category bidder in an asymmetric-auction stage (i.e. $b_{1,\text{Joint}}$ or $b_{2,\text{Joint}}$ in Figure 1) was derived from a single-agent payoff-maximization problem. Furthermore, these two joint-category bids (denoted as $b_{1,\text{Joint}}$ and $b_{2,\text{Joint}}$) were separately submitted by each joint-category bidder; then the auctioneer (i.e. experiment organizer) randomly chose one of them with equal probability (described as 50% and 50% in Figure 1) to be the chosen joint-category bid.

At the result-announcement stage, results within a matched group, including assigned valuations (v_1, v_2, and v_3), aggregated valuation ($\max\{v_1, v_2\}$), bids in each stage (b_1, b_2, b_3, chosen b_{Joint}, and b_{Solo}), and winning/losing statuses in each stage, were announced to the matched group members. However, the identities of bidders were kept hidden.

In addition, monetary payoffs were calculated and added to each participating bidder's account. Monetary payoffs were calculated as follows. After a result-announcement stage, the auctioneer randomly selected (with equal probability) an auction stage in which an outcome was actually paid. Note that since bidders' vNM functions are additively separable, which is usually assumed and accepted in auction literature, this random selection does not affect bidders' payoff-maximization problems in each auction stage.[11] A winner of a symmetric-auction stage was the bidder who submitted the highest bid, and a winning payoff is $v_i - b_i$ where $i \in \{1, 2, 3\}$. In an asymmetric-auction stage, shown in Figure 1, the outcome for a solo-category bidder was determined by comparing b_{solo} and chosen b_{joint}. On the other hand, the outcome for the first joint-category bidder was determined by comparing $b_{1,\text{joint}}$ and b_{solo}, while the outcome of the second joint-category bidder was determined by comparing $b_{2,\text{joint}}$ and b_{solo}.[12] Lastly, at the end of each round, the matched group was dissolved, and participants returned to the pool of bidders.[13]

Regarding the shared information among bidders, the auction results of each round were announced only to matched-group members at the end of each round, and the results of a specific matched group were not available to members of any other group. As a natural consequence, we observe sizable learning and adjusting behavior in the first half of the rounds in each experiment run. For investigating the strategic interactions and estimates of valuations without concern for the learning effect and strategic uncertainty, the rest of this research excludes the data from the first half of the rounds. As the standard structural auction estimation methods deal with bidding data from a non-learning environment, we use the bid data from the second half of each run, in which the the effect of learning behavior is considered to be negligibly small.[14]

As bid data are used (i) for calculating distributional functions and (ii) for constructing compatibility conditions, we need a careful classifications of asymmetric-auction stage bids submitted by joint-category bidders. The first class is called (i) *chosen-then-announced bids*. Bids in this class were chosen by the auctioneer with the 50%-50% random choice rule, then they were announced in a result-announcement stage as described in

[11] This randomized selection process was empirically motivated by the factual observation of timber auctions, in which the U.S. Forest Service randomized different auction rules. Lu and Perrigne (2008) and Athey, Levin, and Seira (2011) exploit such randomization of timber auctions for detailed investigations of identifications and bidding behavior.

[12] We can compare our three-bidder symmetric auctions to the similar three-bidder auctions reported in Bajari and Hortaçsu (2005) (originally reported in Dyer et al. 1989). In their study the, the average observed bid is about 24 percent higher than the average risk-neutral equilibrium bid. In contrast, the average observed bid in the symmetric stage of Chernomaz (2012) is about 16 percent higher than the risk-neutral equilibrium. The three-bidder case is only one of the treatments in the within-subjects designs of both studies. In the other treatments of Bajari and Hortaçsu (2005), the bids are expected to be relatively higher due to an increased number of bidders, while in the other treatments of Chernomaz (2012) bids are expected to be relatively lower due to a reduced number of bidders. The difference in overbidding under different treatments is also consistent with the idea that subjects' bidding errors would be biased in the predicted direction of the additional treatments. Moreover, procedural differences, such as revelations of values at the end of the bidding round to all subjects in Bajari and Hortaçsu (2005), could have important implications for deviations from the equilibrium.

[13] After an asymmetric-auction stage, Chernomaz (2012) further conducted a communication-based asymmetric-auction stage in which within-a-group joint-category bidders were allowed to exchange textual messages via computers. In this study, we eliminate the communication-based asymmetric-auction stage data to focus our analyses on the single-agent payoff maximization problems.

[14] The experimentally assigned valuations (i.e. true valuations) and observed bids are plotted in the Online Appendix section 1.

Figure 2: Venn Diagram of Asymmetric-Auction Stage Joint-Category Bids

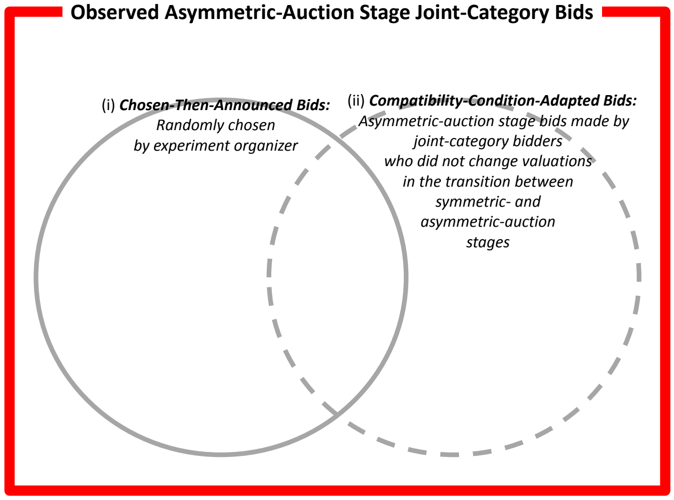


Figure 1. The second class is called (ii) *compatibility-condition-adapted bids*. These bids are made by joint-category bidders whose valuations did change in the transition between symmetric- and asymmetric-auction stages. In Figure 1, a bid made by a joint-category bidder who had a valuation of $v = \max\{v_1, v_2\}$ falls into this class [class (ii)]. Figure 2 and Table 2 explain the categorization and usage of asymmetric-auction stage bid data submitted by joint-category bidders. As the participants were informed of (i) the *chosen-then-announced bids*, we later estimate distributional functions of asymmetric-auction stage joint-category bids based on chosen-then-announced joint-category bids (depicted as the solid-line circle in Figure 2). On the other hand, when we construct compatibility conditions with exogenous variations of auction formats, we use (ii) *compatibility-condition-adapted bids* (depicted as the broken-line circle in Figure 2). This is because compatibility conditions, which will be explained in the estimation section, require a consistent valuation across different auction formats. In contrast to asymmetric-auction stage bids submitted by joint-category bidders, we use all asymmetric-auction stage bids submitted by solo-category bidders both for distributional function estimations and for compatibility condition constructions.

Lastly, Table 3 lists the summary statistics of symmetric-auction stage and asymmetric-auction stage bids.[15] Here, the asymmetric-auction stage bids are *compatibility-condition-adapted* ones on which we later construct compatibility conditions. At the majority of bid quantiles, both joint- and solo-category bidders decreased their bids in asymmetric-auction stages compared to symmetric-auction stages.

[15]In our dataset, similar to other experimental auction studies, overbidding relative to the risk-neutral equilibrium bidding functions is observed (see Table 2 and 3 of Chernomaz 2012), and degrees of risk aversion can in part explain this pattern. In addition, it is worth emphasizing that divergence from fully rational behavior, such as deviations from best responding actions (with various risk attitudes) can be in play as well.

Table 2: Asymmetric-Auction Stage Joint-Category Bid Data Categorization and Their Usage

Class	Description	Usage / Estimating Object
(i) *Chosen-then-announced bids*	Asymmetric-auction stage joint-category bids that were chosen then announced by auctioneer	Distributional functions
(ii) *Compatibility-condition-adapted bids*	Asymmetric-auction stage joint-category bids made by bidders who did not change valuations (in the transition between symmetric- and asymmetric-auction stages)	Compatibility conditions

Note: Non-(chosen-then-announced) bids are discarded in distributional function estimations. Also, asymmetric-auction stage joint-category bids made by the bidders who changed valuations in the transition between symmetric- and asymmetric-auction stages are excluded in the construction of compatibility conditions.

Table 3: Summary Statistics of Bid Data (in U.S. Dollars)

	Bidder Category	Sample Size	Auction Stage	Mean	Standard Deviation	Quantile 10th	25th	50th	75th	90th
Experiment Run I	Joint	192	Symmetric	9.54	3.75	5.00	6.28	9.99	12.40	14.98
		96	Asymmetric	9.58	3.95	4.75	6.76	9.28	12.34	15.00
	Solo	96	Symmetric	6.79	4.60	1.02	2.58	6.69	11.06	12.96
		96	Asymmetric	6.92	4.64	0.75	2.56	7.22	10.93	13.75
Experiment Run II	Joint	144	Symmetric	10.60	4.30	3.16	7.55	11.49	14.18	15.79
		72	Asymmetric	10.22	4.39	3.16	7.38	11.25	13.64	15.63
	Solo	72	Symmetric	8.52	4.88	2.03	4.53	8.61	12.78	15.27
		72	Asymmetric	8.43	4.85	2.03	4.53	8.31	12.75	15.08

Note: Asymmetric-auction stage joint-category bids reported here are *compatibility-condition-adapted bids*.

3 Auction Models

This section describes the models of independent private value (IPV) auctions. We first explain the symmetric auction model that is a straightforward application of Guerre, Perrigne, and Vuong (2000), Guerre, Perrigne, and Vuong (2009), and Campo, Guerre, Perrigne, and Vuong (2011). We then illustrate a simplified version of the asymmetric auction model that was originally proposed by Campo, Perrigne, and Vuong (2003).

3.1 Symmetric Auction Model

A single and indivisible object is sold in an auction to bidders who have the von-Neumann-Morgenstern (vNM) function $U(\cdot)$ that is twice differentiable with $U'(\cdot) > 0$ and $U''(\cdot) \leqslant 0$ to allow potential risk aversion. The normalization of $U(0) = 0$ and $U(1) = 1$ is imposed without loss of generality. In our experiment, there are $N = 3$ bidders in an auction with index $i \in \{1, 2, 3\}$. Each bidder draws a private valuation v_i from the i.i.d. distribution $F_V(v_i)$. Given that other bidders employ a symmetric equilibrium strategy $\phi(\cdot)$, expressed as an inverse bidding function, bidder i's expected payoff maximization problem is $\max_{b_i} U(v_i - b_i) \cdot F_{Y_{-i}}(\phi(b_i))$, where Y_{-i} is a random variable of the highest valuation among opponent bidders with its realization $y_{-i} = \max_{j \neq i} v_j$, and $F_{Y_{-i}}(\cdot)$ is the distribution function of Y_{-i}. We denote $\lambda(\cdot) \equiv U(\cdot)/U'(\cdot)$ and $\lambda^{-1}(\cdot)$ as the corresponding inverse function. We also define B_{-i} as the random variable of the highest bid among opponent bidders with its realization $b_{-i} = \max_{j \neq i} b_j$. In addition, we denote the distribution of B_{-i} as $G_{B_{-i}}(b_{-i})$ and its derivative as $g_{B_{-i}}(b_{-i})$. We henceforth refer to the argument of the mark-down function as the *R factor* or *R function*. Based on Guerre, Perrigne, and Vuong (2009), by denoting the R factor as $R[x|G_{B_{-i}}, g_{b_{-i}}] = G_{B_{-i}}(x)/g_{B_{-i}}(x)$, we can write the first-order necessary condition as $v_i^{\text{Sym}} = b_i^{\text{Sym}} + \lambda^{-1}\left(R^{\text{Sym}}\left[b_i^{\text{Sym}}\middle| G_{B_{-i}}^{\text{Sym}}, g_{B_{-i}}^{\text{Sym}}\right]\right)$, where the superscript of "Sym" emphasizes that the auction model is symmetric.

3.2 Asymmetric Auction Model

We next introduce the asymmetric auction model. We henceforth focus on the simplest environment, a two-category and two-bidder asymmetric auction, on which our experimental asymmetric auction data is based. We define the index of the bidder category as $t \in \{\text{Joint}, \text{Solo}\}$. We use a convenient notation of $-t$ for representing the opponent bidder's category (note: bidders never changed their categories throughout the experiment). A category t bidder draws a private valuation from a distribution $F_{V_t}(v_t)$. Also, we denote $U_t(\cdot)$ as a vNM function of category t bidder, as we allow for the possibility of joint- and solo-category bidders having different payoff functions. In this research, we estimate heterogeneous (yet within-category-homogeneous) payoff functions $U_{\text{Joint}}(\cdot)$ and $U_{\text{Solo}}(\cdot)$ separately. By denoting an inverse bidding function as $\phi_{-t}(\cdot)$, the expected payoff maximization problem is $\max_{b_t} U_t(v_t - b_t) \cdot F_{V_{-t}}(\phi_{-t}(b_t))$, where $F_{V_{-t}}(v_{-t})$ denotes the distribution function of V_{-t}, the valuation of the opponent-category bidder. Simiar to the symmetric case, we used the notations of $\lambda_t(\cdot) \equiv U_t(\cdot)/U_t'(\cdot)$ and $\lambda_t^{-1}(\cdot)$. We denote the distribution of the opponent's bid B_{-t} as $G_{B_{-t}}(b_{-t})$ and its derivative as $g_{B_{-t}}(b_{-t})$. Lastly, by denoting the R factor as $R[x|G_{B_{-t}}, g_{B_{-t}}] = G_{B_{-t}}(x)/g_{B_{-t}}(x)$, we can write the first-order necessary condition as $v_t^{\text{Asym}} = b_t^{\text{Asym}} + \lambda_t^{-1}\left(R^{\text{Asym}}\left[b_t^{\text{Asym}}\middle| G_{B_{-t}}^{\text{Asym}}, g_{B_{-t}}^{\text{Asym}}\right]\right)$, where the superscript of "Asym" emphasizes that the auction model is asymmetric, and which is the simplified version of the framework proposed by Campo, Perrigne, and Vuong (2003).

4 Structural Estimation Methods

In this section, we describe the estimation methods for recovering valuations. Estimation procedures are summarized in three steps: *Step 1* – nonparametrically estimating distribution functions; *Step 2* – by applying semi or nonparametric methods, estimating mark-down functions (i.e. $\lambda_t^{-1}(\cdot)$s); *Step 3* – estimating valuations based on estimated distribution functions and mark-down functions. As the main purpose of this study is to investigate the accuracy of asymmetric auctions estimates, we primarily recover valuations from bids observed in asymmetric auctions, and bid data from symmetric auctions is used solely for the purpose of semi and nonparametrically estimating mark-down functions.

By following the literature, we nonparametrically estimate the distributional and density functions. We use the triweight uni-variate kernel with Silverman's rule of thumb bandwidths. Regarding the bids submitted by joint-category bidders in asymmetric-auction stages, we use the *chosen-then-announced bids* for estimating distributional functions, as these are the bids that were actually announced to the bidders in experimental runs (see Table 2). In addition, we exploit the anonymous nature of our experiment: a bidder did not know opponents' identities in each auction. This enables us to aggregate the distributional functions against which a bidder is best responding. The distributional functions are estimated in a standard fashion. Specifically, we use bandwidths of $h_g^{\text{Sym}} = c_g \cdot (RMN)^{-1/5}$ and $h_g^{\text{Asym}} = c_g \cdot (RM)^{-1/5}$, where R is the number of experiment rounds, M is the number of matched groups, and $N = 3$ is the numbe of bidders. Also, we use $c_g = 3.156 \cdot \hat{\sigma}_b$, and $\hat{\sigma}_b$ is the empirical standard deviation of corresponding observed bids.

The estimated kernel density functions play an important role for calculating R functions. However, the well-known drawback of the standard KDE is the boundary problem. This problem prevents consistent estimation at the boundaries (as well providing underestimations in near-boundary domains) and is a major empirical concern. Indeed, it is observed that the R factor at the right tail of the bid distribution often has quite a large number, as it is generated by dividing a positive number by a number close to zero, tending to result in the inflated estimates of valuations. To overcome the boundary problem, Hickman and Hubbard (2015) applied the method of boundary correction to the structural asymmetric auction estimation framework. Accordingly, throughout this study, we apply the boundary correction methods for all estimates.

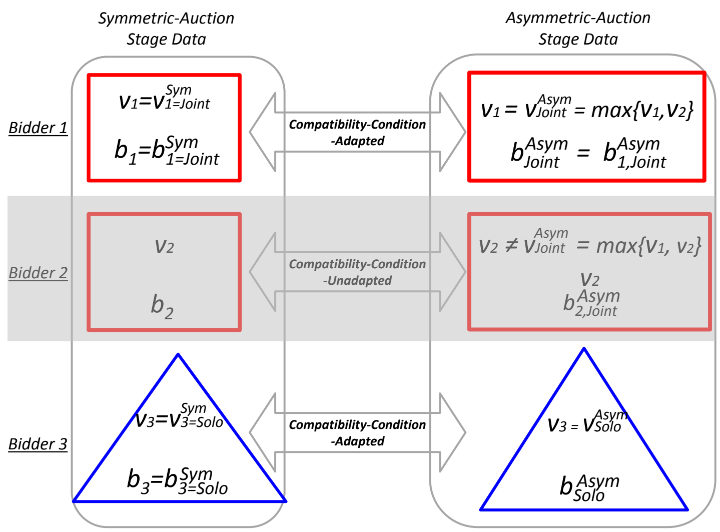
Lastly, as a baseline model, we benchmark the risk-neutral model with the preference of $U_t(x) = x$ and the mark-down function $\lambda_t^{-1}(y) = y$. Accordingly, we have the following estimates $\widehat{v}^{\text{Asym}}_{\text{RN},r,m,t} = b^{\text{Asym}}_{r,m,t} + R^{\text{Asym}}\left[b^{\text{Asym}}_{r,m,t} \middle| \widehat{G}^{\text{Asym}}_{B_{-t}}, \widehat{g}^{\text{Asym}}_{B_{-t}}\right]$ for each category of $t \in \{\text{Joint}, \text{Solo}\}$.

4.1 Semi & Nonparametric Estimations of Risk Averse Models

In empirical auctions, the assumption of risk neutrality is justified when a bidder can be seen as a large firm whose wealth is large relative to the value of an object under auction. However, in reality, bidders could be risk averse, and they may sacrifice extra payments to increase the probability of winning. In such situations derived model implications could be substantially different from those from the risk neutral model.[16] In *Step 2*, we use quantile restrictions to derive *compatibility conditions*, which are used for semi and nonparametrically estimating the mark-down functions. We now introduce the notation for $b^{\text{Sym}}_{i,\alpha}$ to denote the αth quantile for the distribution of observed symmetric-auction stage bids and $b^{\text{Asym}}_{t,\alpha}$ for that of observed asymmetric-auction stage bids submitted by category t bidders. Similarly, we use $v_{i,\alpha}$ as αth quantile of value distribution in symmetric-auction stages and $v_{t,\alpha}$ as that of value distribution among category t bidders in asymmetric-auction stages. Then, we have the quantile notations of the equilibrium first order condition

[16] The empirical evidences and estimates of risk averse preferences in first-price auctions are reported by Lu and Perrigne (2008), Athey, Levin, and Seira (2011), and Campo, Guerre, Perrigne, and Vuong (2011) in their investigations of timber auctions and Campo (2012) in her study of construction contract auctions. Specifically, these empirical investigations emphasize risk averse preferences among small-size firms.

Figure 3: Data Construction for Compatibility Conditions



Note: This figure is a continuation of Figure 1 (Stages within a round). We here assume $v_1 = \max\{v_1, v_2\}$ without loss of generality. Compatibility-condition-*un*adapted bids, like those submitted by Bidder 2 in this figure, are not used for computing bid quantiles used in compatibility conditions.

for the symmetric-auction model as $v^{\text{Sym}}_{i,\alpha} = b^{\text{Sym}}_{i,\alpha} + \lambda^{-1}\left(R^{\text{Sym}}\left[b^{\text{Sym}}_{i,\alpha}\middle| G^{\text{Sym}}_{B_{-i}}, g^{\text{Sym}}_{B_{-i}}\right]\right)$ and for the asymmetric-auction model as $v^{\text{Asym}}_{t,\alpha} = b^{\text{Asym}}_{t,\alpha} + \lambda^{-1}_t\left(R^{\text{Asym}}\left[b^{\text{Asym}}_{t,\alpha}\middle| G^{\text{Asym}}_{B_{-t}}, g^{\text{Asym}}_{B_{-t}}\right]\right)$.

Next, by exploiting the fact that the majority of bidders in experiment runs did not change their valuations in the transition between symmetric- and asymmetric-auction stages mid-round (as depicted in Figure 1), we take advantage of the observed differences between bids across auction stages, which are summarized in Table 3. As the valuations of a solo-category bidder and one of the joint-cagegory bidders were unchanged in both stages, we have the equivalence of valuations, $v^{\text{Sym}}_{i=t} = v^{\text{Asym}}_{t}$ with the notation of $i = t$ for each category of $t \in \{\text{Joint}, \text{Solo}\}$, meaning that we restrict our attention to category t bidders, who ddid not change their valuations. Notably, compatibility-condition-*un*adapted bids (e.g. b^{Sym}_2 and $b^{\text{Asym}}_{2,\text{Joint}}$ submitted by Bidder 2 in Figure 3, whose valuation was changed mid-round) are not used for the construction of the compatibility conditions discussed below. Then, by exploiting the unchanged nature of valuations, we match the quantiles of bidders' private value distribution as $v^{\text{Sym}}_{i=t,\alpha} = v^{\text{Asym}}_{t,\alpha}$, where $v^{\text{Sym}}_{i=t,\alpha}$ and $v^{\text{Asym}}_{t,\alpha}$ denote the αth quantiles of category t bidders' value distribution. Thus, for each category $t \in \{\text{Joint}, \text{Solo}\}$, we can equate the equilibrium first order condition equations. Then, we have the following *compatibility condition* equation:

$$b^{\text{Sym}}_{i=t,\alpha} - b^{\text{Asym}}_{t,\alpha} = \lambda^{-1}_t\left(R^{\text{Asym}}\left[b^{\text{Asym}}_{t,\alpha}\middle| G^{\text{Asym}}_{B_{-t}}, g^{\text{Asym}}_{B_{-t}}\right]\right) - \lambda^{-1}_t\left(R^{\text{Sym}}\left[b^{\text{Sym}}_{i=t,\alpha}\middle| G^{\text{Sym}}_{B_{-i}}, g^{\text{Sym}}_{B_{-i}}\right]\right), \tag{1}$$

where we use the notation of $b^{\text{Sym}}_{i=t,\alpha}$ for the αth quantile of observed bids made by category $i = t$ bidders in symmetric-auction stages. For estimations, we use the data on bid quantiles $\left\{b^{\text{Sym}}_{i=t,\alpha_q}\right\}_{q=0,\cdots,Q}$ and $\left\{b^{\text{Asym}}_{t,\alpha_q}\right\}_{q=0,\cdots,Q}$

that are quantile points of observed bid distributions, and Q is the number of quantile points.[17] Given the quantiles of bid distributions, in *Step 2*, equation (1) can be estimated by the semi and nonparametric methods for obtaining $\widehat{\lambda}_t^{-1}(\cdot)$. Then, in *Step 3*, by plugging the observed bids and estimated objects from *Step 1* and *Step 2* in the first order necessary condition, we can obtain the estimates of valuations as[18] $\widehat{v}^{\text{Asym}} = b^{\text{Asym}} + \widehat{\lambda}_t^{-1}\left(R^{\text{Asym}}\left[b^{\text{Asym}}\middle|\, \widehat{G}_{B_{-t}}^{\text{Asym}}, \widehat{g}_{B_{-t}}^{\text{Asym}}\right]\right)$.

4.1.1 Semiparametric Estimation for CRRA Model

Based on Campo, Guerre, Perrigne, and Vuong (2011), we now apply the preference of constant relative risk averse (CRRA) vNM functions $U_t(x) = x^{\theta_t}$ for $t \in \{\text{Joint}, \text{Solo}\}$. We have the mark-down function, $\lambda_t^{-1}(y) = \theta_t \cdot y$. Then, in *Step 2*, the *compatibility condition* equation (1) with estimated distributional functions ($\widehat{G}$s and $\widehat{g}$s) becomes $\widehat{b}_{i=t,\alpha}^{\text{Sym}} - \widehat{b}_{t,\alpha}^{\text{Asym}} = \theta_t \cdot \left\{\widehat{R}_{B_{-t}}^{\text{Asym}}\left[\widehat{b}_{t,\alpha}^{\text{Asym}}\right] - \widehat{R}_{B_{-i}}^{\text{Sym}}\left[\widehat{b}_{i=t,\alpha}^{\text{Sym}}\right]\right\}$, where we use the shorthand notations for simplicity. Then, with the standard OLS assumptions, we can apply the OLS estimation to this equation for obtaining $\widehat{\theta}_t$. Subsequently, in *Step 3*, we obtain the semiparametric CRRA model estimates of valuations through the first-order condition equation. We apply the bootstrap method for obtaining the standard error of $\widehat{\theta}_t$s.

4.1.2 Nonparametric Estimation Model

Finally, we estimate the nonparametric vNM function model that is introduced by Guerre, Perrigne, and Vuong (2009). Despite the fact that certain classes of payoff functions (e.g. CRRA) have been proposed and applied in empirical research, there is neither a consensus among researchers nor definitive criterion regarding which class of payoff functions accurately describe the bidding behavior. Hence, an extension to nonparametric specification is essential, as it allows the greatest degree in modeling mark-down and payoff functions. Specifically, for nonparametric estimations, Guerre, Perrigne, and Vuong (2009) provide a nonparametric identifiation result based on a recursive construction of quantile points, yet they also indicate the potential problems of serial correlations and accumulated error terms in such a recursive construction. Since a recursive construction is difficult to apply in practice, they suggest sieve as an alternative estimation method for empirical applications.

Based on the alternative estimation method suggested in their research, we use the sieve method to estimate the mark-down functions $\lambda_t^{-1}(\cdot) \in \Lambda^{-1}$, where Λ^{-1} is a set of differentiable and strictly monotonically increasing functions. To the best of our knowledge, this is the first applied auction work using their nonparametric sieve framework for estimating risk averse preferences with observed bid data. In practice, as we normalize a vNM function, we impose the theoretical restrictions of $\lambda_t^{-1}(0) = 0$ and $0 < \frac{d}{dR}\lambda_t^{-1}(R) \leqslant 1$.

[17] See Online Appendix section 2 for a detailed description of quantile point constructions.

[18] Once we estimate $\widehat{\lambda}_t^{-1}(\cdot)$, we can analytically or numerically recover a payoff function $\widehat{U}_t(x)$ by solving the differential equation of $\widehat{\lambda}_t(x) = \widehat{U}_t(x)/\widehat{U}_t'(x)$ with the normalized initial condition of $\widehat{U}(0) = 0$, leading to the solution of $\widehat{U}(x) = \exp\left[\int_1^x 1/\widehat{\lambda}(z)dz\right]$.

As emphasized by Matzkin (1994), Matzkin (2007), and Chen (2007), there are several advantages of such economic-theory-based shape restrictions in empirical analyses, such as credible extrapolations beyond the support of data. Accordingly, we will later exploit shape restrictions. Furthermore, we choose Λ^{-1} as the set of polynomial functions $Pol(y;\eta_{t,\overline{K}}) = \sum_{k=1}^{\overline{K}} \eta_{t,k} \cdot y^k$ without intercept terms, where $\eta_{t,\overline{K}}$ stands for a coefficient vector of $\overline{K}$th order polynomial. In addition, a polynomial order $\overline{K}$ flexibly changes in polynomial sieve space.[19] Then, in *Step 2*, as polynomials are linear in their coefficients, the *compatibility condition* equation (1) becomes

$$\widehat{b}^{\text{Sym}}_{i=t,\alpha} - \widehat{b}^{\text{Asym}}_{t,\alpha} = \sum_{k=1}^{\overline{K}} \eta_{t,k} \left\{ \left(\widehat{R}^{\text{Asym}}_{B_{-t}} \left[\widehat{b}^{\text{Asym}}_{t,\alpha} \right] \right)^k - \left(\widehat{R}^{\text{Sym}}_{B_{-i}} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha} \right] \right)^k \right\}. \tag{2}$$

Under this polynomial specification, Λ^{-1} becomes a linear space, and we can solve the minimization problems by the least square method with the bid quantile data. Additionally, we need a criterion function for selecting the order of polynomial terms. For selecting $\widehat{\eta}_t$ among the estimates of $\left\{ \widehat{\eta}_{t,\overline{K}_{\min}}, \widehat{\eta}_{t,\overline{K}_{\min}+1}, \cdots, \widehat{\eta}_{t,\overline{K}_{\max}-1} \widehat{\eta}_{t,\overline{K}_{\max}} \right\}$, we adopt the Akaike Information Criterion (AIC) (Akaike 1998). Subsequently, in *Step 3*, we obtain the nonparametric estimates of valuations using Equation (2).

For implementation purposes, the empirical challenge of this sieve estimation method is that, in practice, applied researchers are only able to identify $\lambda^{-1}(\cdot)$ on limited domains which are away from boundaries. This limitation mainly comes from two facts: (a) we estimate a nonlinear mark-down function $\lambda^{-1}(\cdot)$, not by applying a nonlinear-recursive-projection estimator but by applying a linear difference estimator based on Equation (2), in which empirical researchers face difficulties in obtaining accurate R factors due to the boundary problem; and more importantly (b) after taking a difference, empirical researchers are able to estimate $\lambda^{-1}(\cdot)$ function on a truncated domain of R factors, while they are technically required to use $\lambda^{-1}(\cdot)$ on the full domain of $[0, R_{\max}]$ for recovering valuations.[20]

To overcome these issues, we use shape restrictions on sieve polynomial estimations, which are frequently suggested in the nonparametric estimation literature.[21] Specifically, conventional-wisdom-based bounds on the slopes of $\lambda_t^{-1}(\cdot)$ functions are employed on boundary domains of R factors. Note that, with these shape restrictions, we are still able to exploit data variations on the domain, where the differences in the right-side of Equation (2) (i.e. $\left(\widehat{R}^{\text{Asym}}_{B_{-t}} \left[\widehat{b}^{\text{Asym}}_{t,\alpha} \right] \right)^k - \left(\widehat{R}^{\text{Sym}}_{B_{-i}} \left[\widehat{b}^{\text{Sym}}_{i=t,\alpha} \right] \right)^k$) are available.

In the next section, we report the sieve estimation results based on the following three types of shape

[19] In programming, we choose the minimum and maximum number of polynomial terms as $\overline{K}_{\min} = 5$ and $\overline{K}_{\max} = 15$ for achieving flexibility in the polynomial sieve space.

[20] See Figure 3 in the Online Appendix section 3.

[21] See Matzkin (2007) pp. 5352 for an example.

restrictions on R functions: (1) minimalistic slope restrictions based only on economic theory (called *minimalistic shape restrictions*), which are $\lambda^{-1}(0) = 0$ and $0 < \frac{d}{dy}\lambda^{-1}(y) \leqslant 1$; (2) shape restrictions based on common lower bounds of slopes among solo- and joint-category bidders (called *common shape restrictions*); and (3) shape restrictions based on differentiated lower bounds of slopes (called *differentiated shape restrictions*) which incorporate heterogeneous risk attitudes and framing effects across bidder categories that are frequently reported in both the empirical and experimental auction literature.[22] The technical note in the Online Appendix section 3 provides the details and computational implementations of these shape restrictions.

5 Estimation and Test Results

This section reports the results of estimations and statistical tests under various modeling assumptions. As the purpose of this study is investigating the accuracy of asymmetric auction estimates, we analyze estimates of valuations derived from the asymmetric models. First, we graphically describe laboratory-assigned true valuations and estimated valuations. Then, we statistically test their distributional equivalence. Regarding the model specifications on vNM payoff functions, we start with the risk neutral model. Then, we discuss risk-averse models starting with CRRA, followed by nonparametric models with various shape restrictions. Lastly, we compare estimated and true market efficiencies. The Online Appendix section 4 and 5 list L^1 and L^2 distance norms between estimated and true valuations, as well as estimated density functions.

5.1 Estimation Results

The estimation results are plotted in Figures 4 to 8, which depict laboratory-assigned true valuations on the horizontal axis and estimated valuations on the vertical axis. For measuring the deviations from true valuations, a 45-degree line is added. Also, for creating equally-scaled figures (so that a 45-degree line is tilted exactly at 45 degrees), the estimated valuations are censored from above at \$30. Note that a few of the estimated valuations, especially ones derived from the risk neutral model, exceed \$100.

First, we discover the severe over-estimation of asymmetric auction estimates with the risk neutral model depicted in Figure 4. Thus, our result confirms a similar finding detected in the symmetric auction model reported by Bajari and Hortaçsu (2005); the existence of asymmetry among bidders does not change the over-estimation of the risk neutral model.

The estimated valuations derived from the semiparametric CRRA model are plotted in Figure 5. Also, the OLS estimates of risk averse parameters, θ_ts, are reported in Table 4 with 95% bootstrapped confidence

[22]See the empirical evidence of Campo (2012) and the experimental investigation of Holt and Laury (2002) for such heterogeneous risk preferences and framing effects.

intervals.[23] For both categories of bidders, and in both experiment runs, the degrees of the Arrow-Pratt relative risk measures $(1-\theta_t)$, revealed by submitted bids, are remarkably large in this laboratory experiment, an observation that is frequently reported in other experimental auction studies. Our results report pros and cons of the semiparametric estimation method. As pros, the problem of over-estimation is largely resolved, and estimated valuations among solo-category bidders lie relatively close to the 45 degree line. However, we now also have noticeable systematic under-estimation among joint-category bidders.[24] In addition, the 95% bootstrapped confidence intervals of CRRA parameter are wide, which indicates the unstable nature of resulting valuation estimates. The Online Appendix section 5 reports the density estimates based on the CRRA model estimated valuations with 95% sub-sampled bounds, and this instability is manifested by the wide subsampled bounds.

Lastly, we progress to the nonparametric sieve estimation results. The estimates are plotted in Figure 6 (with *minimalistic shape restrictions* based on auction theory), Figure 7 (with conventional-wisdom-based *common shape restrictions*), and Figure 8 (with *differentiated shape restrictions*). We observe that the problem of systematic under-estimation among joint-category bidders is improved, supporting the empirical usefulness of the shape-restricted nonparametric sieve estimation method, assessed by L^1 measure, listed in the Online Appendix section 4.

5.2 True and Estimated Valuations - Test for Distributional Equivalence

In this subsection, to test the statistical equivalence between true and estimated value distributions, we use the Modified Kolmogorov-Smirnov statistic (henceforth MKS) that is proposed by Haile, Hong, and Shum (2003) and extended by Bajari and Hortaçsu (2005).

The Modified Kolmogorov-Smirnov statistic for category $t \in \{\text{Joint}, \text{Solo}\}$ bidders is calculated as follows:

$$\text{MKS}_t = \sqrt{RM} \sup_{v \in [v_\star, v^\star]} \left| \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \mathbb{1}\left\{\widehat{v}_{r,m,t} \leqslant v\right\} - F_t(v) \right|,$$

where $F_t(v)$ is a known distribution of drawn valuations in our experimental auction data. As a reminder, $r \in \{1, 2, \cdots, R\}$ is an auction round index and $m \in \{1, 2, \cdots, M\}$ is a within-a-round matched group index. Also, $v_\star$ and $v^\star$ are strictly bounded away from boundaries. As the asymptotic normality of MKS statistics is not yet known in the literature, and based on the proofs provided by Haile, Hong, and Shum (2003) and

[23] In Table 4, we list the estimation results with boundary corrections. The estimation results without boundary corrections can be provided upon request. Regarding the bootstrap, we re-sample 1000 times for calculating confidence intervals. Note that, if we affine-transform the CRRA vNM function into the well-known form of $U(x)=\frac{x^{1-\nu}-1}{1-\nu}$, the negative values in the 95% confidence bounds could be interpreted as extreme risk aversion.

[24] Such systematic under-estimation is also found in semiparametric estimates of symmetric auctions in Bajari and Hortaçsu (2005) when the number of bidders is large.

Bajari and Hortaçsu (2005), we consider subsampling by using the following smooth analogue for computing the nondegenerate asymptotic distribution,[25]

$$\widetilde{\mathrm{MKS}}_t = \sqrt{RM} \sup_{v \in [v_\star, v^\star]} \left| \frac{1}{RM} \sum_{r=1}^{R} \sum_{m=1}^{M} \Lambda\left(\widehat{v}_{r,m,t} - v\right) - F_t(v) \right|,$$

where $\Lambda(x) = 1 - \psi(\frac{x}{h})$ with a bandwidth parameter h and $\psi(\cdot)$ is a smooth and strictly monotonic distribution function. The auction econometrics literature has proven uniform consistencies under various models on the support that is strictly bounded away from boundaries.[26] Accordingly, under the null of a correct model specification, the sup distance between smoothed and true distribution converges to zero.

For generating an approximate distribution, we draw S subsamples from the observed bids. Theoretically speaking, we could draw all combinatorics of $\kappa_S = {}_{RM}\mathbb{C}_S$ unique subsamples. However, in computation, it is not practically feasible to compute all ${}_{RM}\mathbb{C}_S$ combinations. For this reason, we set the subsampling draw size of $\kappa_S = 600$.[27] The subsampling distribution $\Phi_t(x)$ for $\widetilde{\mathrm{MKS}}_t$ of category $t \in \{\text{Joint}, \text{Solo}\}$ is approximated by

$$\Phi_t(x) = \frac{1}{\kappa_S} \sum_{l=1}^{\kappa_S} \mathbb{1} \left\{ \sqrt{S} \sup_{v \in [v_\star, v^\star]} \left| \frac{1}{S} \sum_{s=1}^{S} \Lambda\left(\widehat{v}^{l}_{s,t} - v\right) - F_t(v) \right| \leqslant x \right\}.$$

To increase statistical power, we further aggregate the estimated valuations after each method of structural estimation (denoted as "Aggregated Run" in Table 5), and analyze the test results of the Aggregated Run.

The test results are listed in Table 5 with a five percent significance level. In the table, "YES" displays that the distributional equivalence is rejected, while "NO" indicates non-rejection at the significance level of five percent. At the Aggregate Run level, the distributional equivalence in the risk neutral model is strongly rejected for joint-category bidders at the one percent level but not rejected for solo-category bidders at the ten percent level. However, the p-values of risk-neutral model solo-category estimates within each experiment run (0.112 and 0.103) are close to the ten percent rejection level, suggesting that empirical researchers should consider the usage of more advanced risk-preference models. Next, by applying the CRRA semiparametric model, we fail to reject the distributional equivalence among solo-category bidders. On the other hand, although they are improved when compared to those in the risk neutral models, estimates among joint-category bidders still do not attain equivalence at the ten percent, indicating the need for further improvements. Lastly,

[25] See the Appendix of Bajari and Hortaçsu (2005) for the proof of non-degenerating asymptotic distribution.

[26] Note that $\widetilde{\mathrm{MKS}}_t \to \mathrm{MKS}_t$ as $h \to 0$. As in Bajari and Hortaçsu (2005), we choose $h = 1$ in our subsampling computation. Also, we set the domain of sup operator as $[v_\star, v^\star] = [\$4.6875, \$16.237]$ where $v_\star = \$4.6875$ corresponds to the 25th quantile of solo-category bidder value distribution, while $v^\star = \$16.237$ approximately corresponds to the 75th quantile of joint-category bidder value distribution.

[27] In computation, we set $S = 36$ for Experiment Run I and $S = 27$ for Run II, which are 37.5% of RM in each experiment run. Politis, Romano, and Wolf (1999) prove that, under the null, if $S \to \infty$ and $\frac{S}{RM} \to 0$ as $RM \to \infty$, this approximation provides a consistent estimate of true sampling distribution.

by applying the nonparametric sieve estimations, at the Aggregate Run level, we fail to reject distributional equivalences, for both bidder categories, at the five percent level when conventional-wisdom-based *common shape restrictions* and *differentiated shape restrictions* are applied. Thus, conventional-wisdom-based shape restrictions are empirically shown to help to achieve higher accuracy.

In summary, these distributional equivalence tests confirm that researchers attain higher accuracy when they use more advanced semi and nonparametric econometrics, providing an experimental support for the auction estimation methods developed in recent years.

5.3 Efficiency Assessment of Asymmetric Auctions

In empirical asymmetric auction research, the assessment of efficiency in an auction market is of central interest for market design. The key question is whether efficiency recovered from the structural estimations is useful in practice. To the best of our knowledge, our study is the first to report the accuracy in structurally estimated auction market efficiencies.

Table 6 summarizes and compares the true and estimated efficiencies with the boundary correction method. The table shows that the recovered efficiencies are within 2.5% difference from the true efficiency (at the Aggregated Run level) in the risk-neutral, nonparametric *common shape restrictions*, and nonparametric *differentiated shape restriction* models. As a note, CRRA and nonparametric *minimalistic shape restriction* models underestimate the efficincies, mostly due to the underestimation of valuations among joint-category bidders as shown in Figures 5 and 6. The new and somewhat surprising finding here is that the primitive risk neutral model, which does not require exogenous variations, recovers the accuracy within 2.5% range from true efficiency at the Aggregated Run level. These findings confirm the empirical usefulness of asymmetric structural auction methods for the purpose of assessing market efficiency.

Figure 4: Risk Neutral Model

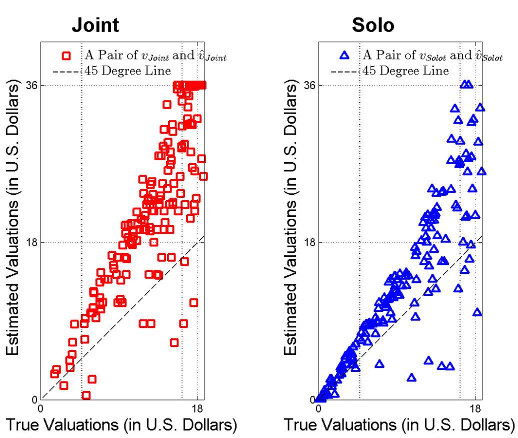


Figure 5: Semiparametric CRRA Model

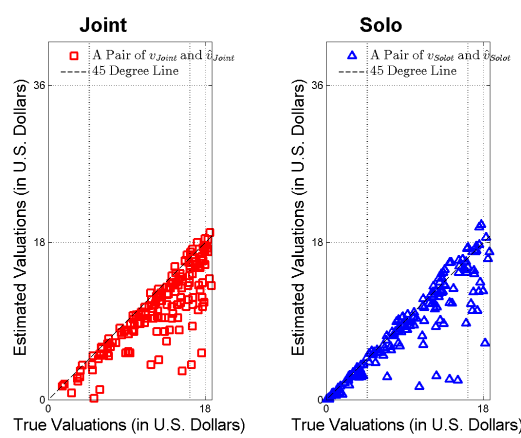


Figure 6: Nonparametric Model -
Minimalistic Shape Restrictions

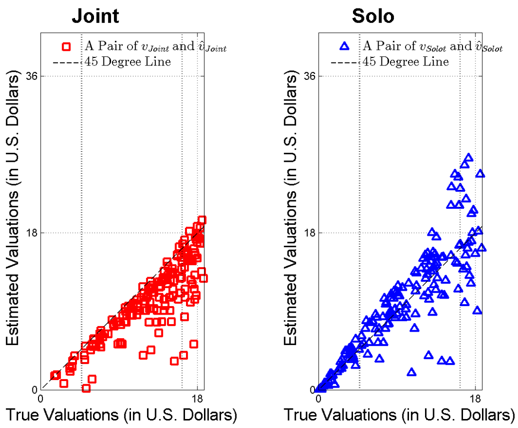


Figure 7: Nonparametric Model -
Common Shape Restrictions

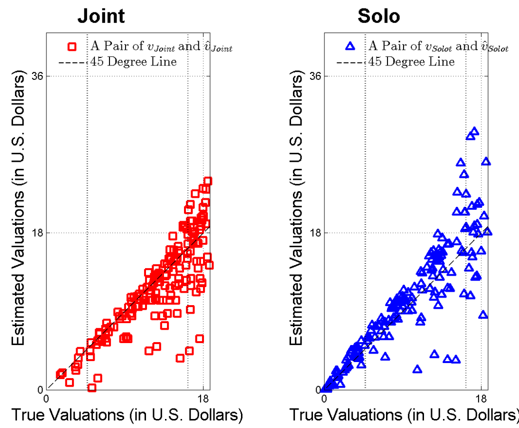


Figure 8: Nonparametric Model -
Differentiated Shape Restrictions

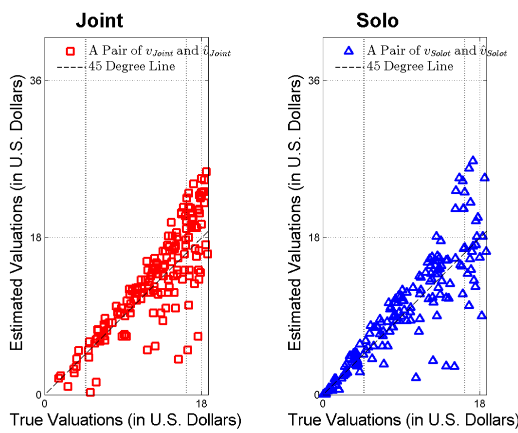


Table 4:
OLS Estimates of CRRA Risk-Averse Parameters ($U_t(x) = x^{\theta_t}$)
with Boundary Corrections

	Bidder Category	CRRA: $\hat{\theta}_t$	
Experiment Run I	Joint	0.053	(-0.051, 0.199)
	Solo	0.060	(-0.794, 0.455)
Experiment Run II	Joint	0.040	(-0.041, 0.190)
	Solo	0.173	(-0.192, 0.338)

Note: The parentheses indicate bootstrapped 95 % confidence intervals.

Table 5: True versus Estimated Valuations - Test for Distributional Equivalence
(Reject Distributional Equivalence at 5% Level? YES (Rejected); NO (Not Rejected)

	Bidder Category	Risk Neutral	Semiparametric CRRA	Nonparametric: *Minimal. Shape Restrictions*	Nonparametric: *Common Shape Restrictions*	Nonparametric: *Differen. Shape Restrictions*
Experiment Run I	Joint	4.899*** (0.000) YES	2.960*** (0.008) YES	3.266*** (0.000) YES	1.939** (0.043) YES	1.021 (0.275) NO
	Solo	1.939 (0.112) NO	1.531 (0.525) NO	0.816 (0.700) NO	0.816 (0.490) NO	0.816 (0.678) NO
Experiment Run II	Joint	3.182*** (0.002) YES	2.239** (0.018) YES	2.357*** (0.002) YES	1.061 (0.440) NO	0.825 (0.497) NO
	Solo	2.239 (0.103) NO	0.825 (0.555) NO	0.943 (0.453) NO	0.589 (0.867) NO	0.825 (0.578) NO
Aggregated Run (Run I and II, aggregated after value estimations)	Joint	5.786*** (0.000) YES	3.163*** (0.007) YES	3.472*** (0.000) YES	2.006* (0.052) NO	0.694 (0.717) NO
	Solo	2.623* (0.052) NO	1.620 (0.450) NO	0.694 (0.802) NO	0.772 (0.607) NO	0.694 (0.773) NO

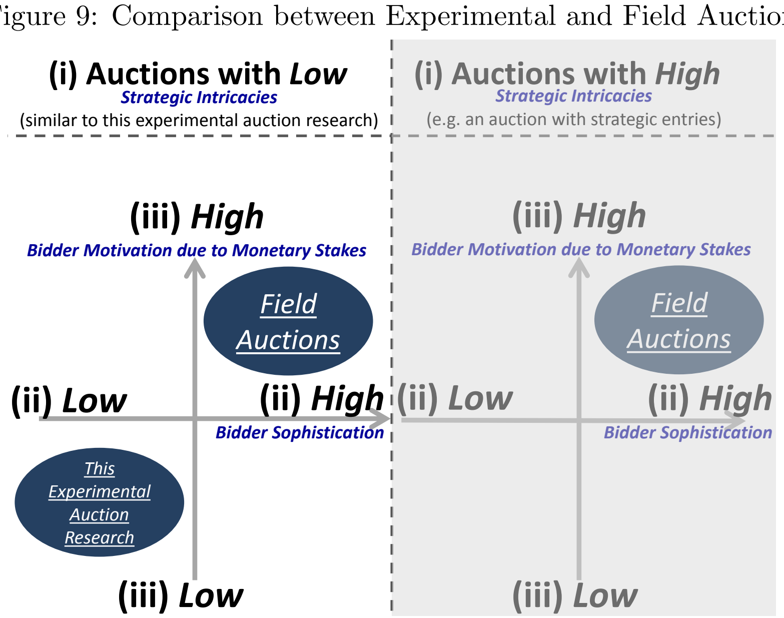
The parentheses indicate a p-value. Modified Kolmogorov-Smirnov statistics are calculated on [$4.6875, $16.237].
*** indicates the rejection at 1%, ** at 5%, and * at 10%.

Table 6: Assessment of True versus Estimated Efficiencies

	True Efficiency	Risk-Neutral	Semiparametric CRRA	Nonparametric: *Minimal. Shape Restrictions*	Nonparametric: *Common Shape Restrictions*	Nonparametric: *Differen. Shape Restrictions*
Experiment Run I	85.42%	89.58% (+4.17%)	76.04% (-9.38%)	75.00% (-10.42%)	81.25% (-4.17%)	84.37% (-1.04%)
Experiment Run II	90.28%	90.28% (±0.00%)	81.94% (-8.33%)	81.94% (-8.33%)	91.67% (+1.39%)	95.83% +(5.56%)
Aggregated Run (Run I and II, aggregated after value estimations)	87.50%	89.88% (+2.38%)	78.57% (-8.93%)	77.98% (-9.52%)	85.71% (-1.79%)	89.29% (+1.79%)

The parentheses indicate the gap from the true auction market efficiency.

Figure 9: Comparison between Experimental and Field Auctions



6 Conclusion

To address the precision of structural auction estimates, this research provides laboratory evidence to support the accuracy of asymmetric first-price auction estimates. We provide new statistical evidence by conducting distributional equivalence tests between the estimated and true valuations. Also, we show that the usage of risk averse semi and nonparametric estimation methods leads to nonnegligible improvements in asymmetric auction estimates, and that advanced estimation techniques, in general, achieve higher accuracy. Although the *degree* of such improvement will differ by applications, the *fact* of improvement is generalizable to other empirical auction research.

Finally, the external validity of our results to other auctions must be addressed. We acknowledge that getting a good estimation result in one specific auction environment does not guarantee that researchers will get a similar result in other situations. However, one can deduce conservative yet practical insights by contrasting our experimental auctions with field auctions. Experimental auctions differ from field ones in at least three ways: (i) the strategic intricacies of auctions; (ii) bidder sophistication; and (iii) bidder motivation due to monetary stakes. The discussion below, as depicted in Figure 9, breaks down (i) into two parts (i.e. low and high strategic intricacies compared to this research), then examines the generalizability of our results regarding (ii) and (iii).

Asymmetric auctions with high/advanced strategic intricacies (right hand side of Figure 9, shaded area): If an environment of empirical asymmetric auction research is more intricate than the one we have discussed in this study, such as endogenous and strategic participation in auctions or binding reserve prices, our results have restricted external validity on the accuracy of estimates. Bidders who face such high degrees of strategic intricacies may behave differently from what we observe in our experiment. Further investigation on the accuracy of estimates derived from experimental data, or any field data that directly or indirectly contains information about underlying valuations, will extend the results of this study for such auctions.

Asymmetric auctions with low (or similar) strategic intricacies (left hand side of Figure 9): In our experimental asymmetric auctions, the participants were undergraduate students who lack real-world business experience, so their degree of strategic sophistication is reasonably assumed to be lower than bidders in the real-world business industry (i.e. low degree of (ii)). In addition, as the monetary stakes in our experiment are relatively low compared to the stakes observed in real-world auctions, the associated monetary motivation among bidders in the laboratory is also expected to be low (i.e. low degree of (iii)). However, the positive finding of this research is that the structural estimates derived from bids, submitted by strategically unsophisticated and less financially motivated bidders, are statistically shown to be distributionally equivalent. Therefore, we can deductively translate the accuracy of estimates reported in this research into the estimates generated from professional industry bidders in field auctions for the following reasons: first, professional bidders must have a high degree of strategic sophistication in order to survive industry competition (i.e. high degree of (ii)), and secondly, as monetary stakes in real-world auctions are high, the associated monetary motivations among industry bidders are also high (i.e. high degree of (iii)). It stands to reason that, compared to our experiment participants, such professional bidders are more likely to recognize underlying strategic interactions in auctions as prescribed by BNE and less likely to make optimization errors (or less likely to engage in irrational behavior). Accordingly, because structural estimations are based on BNE, the estimates derived from such sophisticated and considered bids are likely to be more accurate than those reported in this research. Thus, we deduce that, as long as the strategic intricacy of an underlying asymmetric auction market is not vastly different from the one discussed in this research, and as long as industry bidders are maximizing expected payoffs, what holds accurate in our laboratory auctions also holds accurate in a real-world industry setting.

Lastly, we end this study by suggesting an avenue for further research. Due to their direct relevance to market designs, empirical auction studies with entry decisions among potentially risk averse bidders are an important area and are currently being actively investigated (see Gentry and Li 2014, Li et al. 2015, Gentry et al. 2015, and Kong 2015). Another potential extension is learning, which is also actively researched these days (see Nekipelov 2007 and Doraszelski et al. 2018), and which warrants further investigation. If researchers observe long auction series with either true or proxy valuations, the modeling contribution of the learning process could be explored. Experimental or field-experimental verifications of entry and learning models could provide a solid foundation for auction market policy designs.

References

Akaike, Hirotogu (1998), "Information theory and an extension of the maximum likelihood principle." In *Selected Papers of Hirotugu Akaike*, 199–213, Springer.

Asker, John (2010), "A study of the internal organization of a bidding cartel." *American Economic Review*, 100, 724–762.

Athey, Susan, Jonathan Levin, and Enrique Seira (2011), "Comparing open and sealed bid auctions: Evidence from timber auctions." *Quarterly Journal of Economics*, 126, 207–257.

Bajari, Patrick and Ali Hortaçsu (2005), "Are structural estimates of auction models reasonable? Evidence from experimental data." *Journal of Political Economy*, 113, 703–741.

Bajari, Patrick and Lixin Ye (2003), "Deciding between competition and collusion." *Review of Economics and statistics*, 85, 971–989.

Balat, Jorge (2012), "Highway procurement and the stimulus package: Identification and estimation of dynamic auctions with unobserved heterogeneity." Technical report, Working paper.

Campo, Sandra (2012), "Risk aversion and asymmetry in procurement auctions: Identification, estimation and application to construction procurements." *Journal of Econometrics*, 168, 96–107.

Campo, Sandra, Emmanuel Guerre, Isabelle Perrigne, and Quang Vuong (2011), "Semiparametric estimation of first-price auctions with risk-averse bidders." *Review of Economic Studies*, 78, 112–147.

Campo, Sandra, Isabelle Perrigne, and Quang Vuong (2003), "Asymmetry in first-price auctions with affiliated private values." *Journal of Applied Econometrics*, 18, 179–207.

Cantillon, Estelle (2008), "The effect of bidders' asymmetries on expected revenue in auctions." *Games and Economic Behavior*, 62, 1–25.

Chen, Xiaohong (2007), "Large sample sieve estimation of semi-nonparametric models." *Handbook of econometrics*, 6, 5549–5632.

Chernomaz, Kirill (2012), "On the effects of joint bidding in independent private value auctions: An experimental study." *Games and Economic Behavior*, 76, 690–710.

De Silva, Dakshina G, Timothy Dunne, and Georgia Kosmopoulou (2003), "An empirical analysis of entrant and incumbent bidding in road construction auctions." *Journal of Industrial Economics*, 51, 295–316.

Donald, Stephen G and Harry J Paarsch (1993), "Piecewise pseudo-maximum likelihood estimation in empirical models of auctions." *International Economic Review*, 121–148.

Donald, Stephen G and Harry J Paarsch (1996), "Identification, estimation, and testing in parametric empirical models of auctions within the independent private values paradigm." *Econometric Theory*, 517–567.

Doraszelski, Ulrich, Gregory Lewis, and Ariel Pakes (2018), "Just starting out: Learning and equilibrium in a new market." *American Economic Review*, 108, 565–615.

Dyer, Douglas, John H Kagel, and Dan Levin (1989), "Resolving uncertainty about the number of bidders in independent private-value auctions: an experimental analysis." *The RAND Journal of Economics*, 268–279.

Elyakime, Bernard, Jean Jacques Laffont, Patrice Loisel, and Quang Vuong (1994), "First-price sealed-bid auctions with secret reservation prices." *Annales d'Economie et de Statistique*, 115–141.

Ertaç, Seda, Ali Hortaçsu, and James W Roberts (2011), "Entry into auctions: An experimental analysis." *International Journal of Industrial Organization*, 29, 168–178.

Flambard, Véronique and Isabelle Perrigne (2006), "Asymmetry in procurement auctions: Evidence from snow removal contracts." *Economic Journal*, 116, 1014–1036.

Gentry, Matthew and Tong Li (2014), "Identification in auctions with selective entry." *Econometrica*, 82, 315–344.

Gentry, Matthew L, Timothy P Hubbard, Denis Nekipelov, and Harry J Paarsch (2018), "Structural econometrics of auctions: a review." *Foundations and Trends® in Econometrics*, 9, 79–302.

Gentry, Matthew L, Tong Li, and Jingfeng Lu (2015), "Identification and estimation in first-price auctions with risk-averse bidders and selective entry." *Available at SSRN 2587812*.

Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong (2000), "Optimal nonparametric estimation of first-price auctions." *Econometrica*, 68, 525–574.

Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong (2009), "Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions." *Econometrica*, 77, 1193–1227.

Haile, Philip A, Han Hong, and Matthew Shum (2003), "Nonparametric tests for common values at first-price sealed-bid auctions." Technical report, National Bureau of Economic Research.

Hendricks, Kenneth, Joris Pinkse, and Robert H Porter (2003), "Empirical implications of equilibrium bidding in first-price, symmetric, common value auctions." *Review of Economic Studies*, 70, 115–145.

Hendricks, Kenneth and Robert H Porter (1988), "An empirical study of an auction with asymmetric information." *American Economic Review*, 865–883.

Hendricks, Kenneth and Robert H Porter (1992), "Joint bidding in federal ocs auctions." *American Economic Review*, 82, 506–511.

Hendricks, Kenneth, Robert H Porter, and Charles A Wilson (1994), "Auctions for oil and gas leases with an informed bidder and a random reservation price." *Econometrica: Journal of the Econometric Society*, 1415–1444.

Hickman, Brent R and Timothy P Hubbard (2015), "Replacing sample trimming with boundary correction in nonparametric estimation of first-price auctions." *Journal of Applied Econometrics*, 30, 739–762.

Hickman, Brent R, Timothy P Hubbard, and Yiğit Sağlam (2012), "Structural econometric methods in auctions: A guide to the literature." *Journal of Econometric Methods*, 1, 67–106.

Holt, Charles A and Susan K Laury (2002), "Risk aversion and incentive effects." *American economic review*, 92, 1644–1655.

Jofre-Bonet, Mireia and Martin Pesendorfer (2003), "Estimation of a dynamic auction game." *Econometrica*, 71, 1443–1489.

Kessler, Judd and Lise Vesterlund (2015), *The external validity of laboratory experiments: The misleading emphasis on quantitative effects*, volume 18. Oxford University Press Oxford, UK.

Kong, Yunmi (2015), "Endogenous entry and risk aversion in oil and gas lease auctions." *Manuscript. Department of Economics. New York University.*

Krasnokutskaya, Elena and Katja Seim (2011), "Bid preference programs and participation in highway procurement auctions." *American Economic Review*, 101, 2653–2686.

Laffont, Jean-Jacques, Herve Ossard, and Quang Vuong (1995), "Econometrics of first-price auctions." *Econometrica: Journal of the Econometric Society*, 953–980.

Li, Tong, Jingfeng Lu, and Li Zhao (2015), "Auctions with selective entry and risk averse bidders: theory and evidence." *RAND Journal of Economics*, 46, 524–545.

Lu, Jingfeng and Isabelle Perrigne (2008), "Estimating risk aversion from ascending and sealed-bid auctions: the case of timber auction data." *Journal of Applied Econometrics*, 23, 871–896.

Marion, Justin (2007), "Are bid preferences benign? the effect of small business subsidies in highway procurement auctions." *Journal of Public Economics*, 91, 1591–1624.

Matzkin, Rosa L (1994), "Restrictions of economic theory in nonparametric methods." *Handbook of Econometrics*, 4, 2523–2558.

Matzkin, Rosa L (2007), "Nonparametric identification." *Handbook of Econometrics*, 6, 5307–5368.

Nekipelov, Denis (2007), "Entry deterrence and learning prevention on ebay." *Manuscript, Duke University.*

Paarsch, Harry J (1992), "Deciding between the common and private value paradigms in empirical models of auctions." *Journal of Econometrics*, 51, 191–215.

Pesendorfer, Martin (2000), "A study of collusion in first-price auctions." *Review of Economic Studies*, 67, 381–411.

Politis, Dimitris, Joseph P Romano, and Michael Wolf (1999), "Weak convergence of dependent empirical measures with application to subsampling in function spaces." *Journal of statistical planning and inference*, 79, 179–190.

Porter, Robert H and J Douglas Zona (1999), "Ohio school milk markets: An analysis of bidding." *RAND Journal of Economics*, 263–288.

Salz, Tobias and Emanuel Vespa (2015), "Estimating dynamic games of oligopolistic competition: An experimental investigation." *Available at SSRN 2621270*.